Proofs: Mathematical Induction

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Situation: Suppose you have an infinite sequence of statements S_1 , S_2 , S_3 , … indexed by the natural numbers

Mathematical Induction: To prove all of these statements are true…

1) Base Case: Prove the first statement S_1 is true

2) Induction Step: Prove $\forall\ k \in \mathbb{N}$, if S_k is true, then S_{k+1} is true

Then you have proven that all of the statements S_1 , S_2 , S_3 , … are true

Note: The starting index doesn't have to be 1.
It could be any integer
(Most common starting indexes are 1, 0, or 2)

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Ex 1: Prove $\forall$ integers $n \geq 0$, $n^2 + n$ is even

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Ex 2: Prove $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all integers $n \geq 1$

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Ex 3: Prove $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

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Ex 4: Prove $\forall\ n \in \mathbb{N}$, $5^n - 2^n$ is a multiple of 3